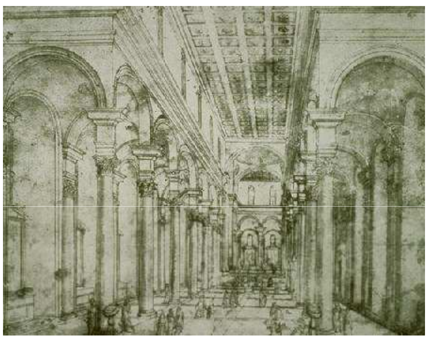
geometric camera model

image acquisition

600 years ago



church of the Holy Spirit, Brunelleschi, 1436

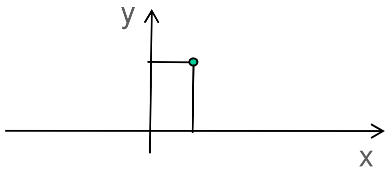
today

the camera projects 3D points
into an image plane

geometric primitives and transformations

geometric primitives: 2D points

Cartesian coordinates $x = (x, y) = \begin{bmatrix} x \\ y \end{bmatrix} \in IR^2$

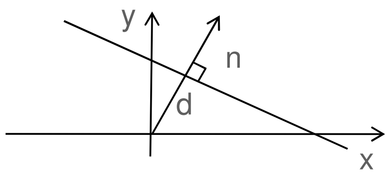


homogeneous coordinates $\tilde{x} = \lambda\,(x, y, 1) = \lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \in IR^3 \setminus \{0\} \qquad \lambda \neq 0$

the same 2D point can be represented by several (infinite) homogeneous vectors.

Points of the form x=(a,b,0) are called points at infinity since they do not correspond to (finite) 2D points.

geometric primitives: 2D lines



2D line $\tilde{x} \cdot \tilde{l} = ax + by + c = 0$

normalization: $\tilde{l} = (\hat{n}, d)$ $\hat{n}$ - normal vector $(\|\hat{n}\| = 1)$

d - distance to the origin

line at infinity $\tilde{l} = (0,0,1)$ (includes all points at infinity)

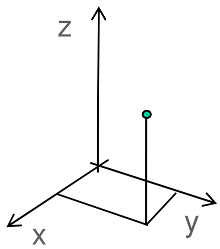
intersection of two lines

$$\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$$

line joining two points

$$\tilde{l} = \tilde{x}_1 \times \tilde{x}_2$$

geometric primitives: 3D points



Cartesian coordinates $x = (x, y, z) = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in IR^3$

homogeneous coordinates $\tilde{x} = \lambda\,(x, y, z, 1) = \lambda \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \in IR^4 \setminus \{0\} \qquad \lambda \neq 0$

the same 2D point can be represented by several (infinite) homogeneous vectors.

Points of the form x=(a,b,c,0) are called points at infinity since they do not correspond to (finite) 2D points.

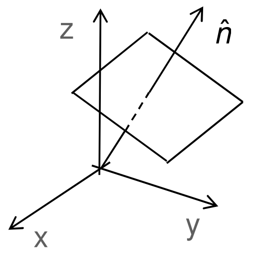
geometric primitives: 3D lines and planes

plane

$$\tilde{x}\cdot\tilde{l} = ax + by + cy + d = 0$$

$$\tilde{l} = (\hat{n}, d)$$ $\hat{n}$ - normal vector $(\|\hat{n}\| = 1)$

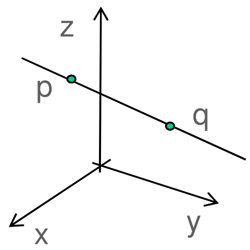
d - distance to the origin



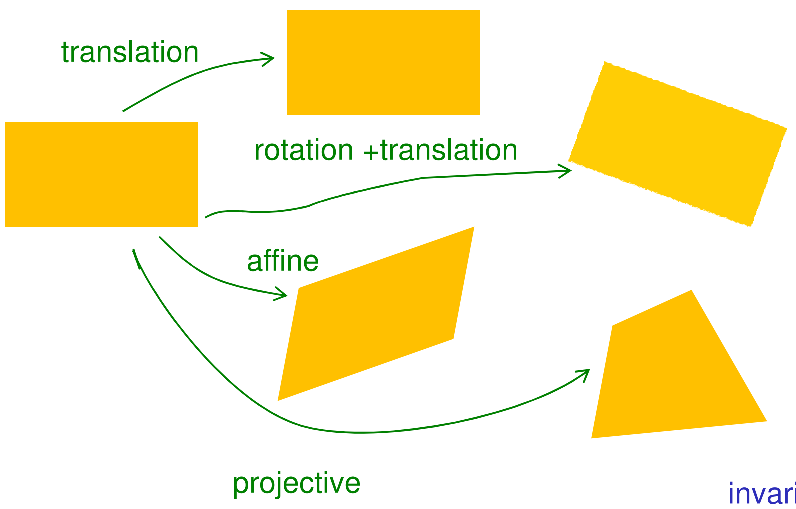
line

$$x = \alpha p + (1-\alpha) q$$ p, q - two points

$$\tilde{x} = \alpha\,\tilde{p} + \beta\,\tilde{q}$$



geometric transformations in the plane



invariance

	lines	paralelism	length	orientation
T	Y	Y	Y	Y
T+R	Y	Y	Y	N
A	Y	Y	N	N
P	Y	N	N	N

transformation of points in the plane

translation $x' = x + t$ $\quad \tilde{x}' = \begin{bmatrix} I & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$

rotation +translation $x' = Rx + t$ $\quad \tilde{x}' = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$ $\quad R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

affine $x' = Ax + t$ $\quad \tilde{x}' = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$ $\quad A = \begin{bmatrix} a_{00} & a_{01} \\ a_{11} & a_{11} \end{bmatrix}$

projective

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}}$$

$$y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$

$$\tilde{x}' = H\tilde{x}$$

transformation of lines

Previous transformations can be written as $\tilde{x}' = H\tilde{x}$

can we use this equation to transform lines? $\tilde{l}^T \tilde{x} = 0$

$$\tilde{l}^T \tilde{x} = \tilde{l}^T H^{-1} \tilde{x}' = \tilde{l}'^T \tilde{x}' = 0$$

this is the equation of a line in the transformed space with

$$\tilde{l}' = H^{-T} \tilde{l}$$

transformation of points in 3d space

translation $x' = x + t$ $\tilde{x}' = \begin{bmatrix} I & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$

rotation +translation $x' = Rx + t$ $\tilde{x}' = \begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$ $\mathrm{R}^\mathrm{T} R = RR^\mathrm{T} = I$

affine $x' = Ax + t$ $\tilde{x}' = \begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix} \tilde{x}$ $\mathrm{A} = \begin{bmatrix} a_{00} & a_{01} \\ a_{11} & a_{11} \end{bmatrix}$

projective

$$x' = \frac{h_{00}x + h_{01}y + h_{02} + h_{03}}{h_{30}x + h_{31}y + h_{32} + h_{33}}$$

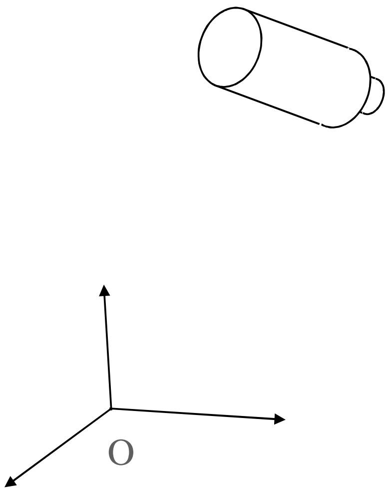
$$y' = \frac{h_{10}x + h_{11}y + h_{12} + h_{12}}{h_{30}x + h_{31}y + h_{32} + h_{33}}$$

$$y' = \frac{h_{20}x + h_{21}y + h_{22} + h_{23}}{h_{30}x + h_{31}y + h_{32} + h_{33}}$$

$$\tilde{x}' = H\tilde{x}$$

camera model

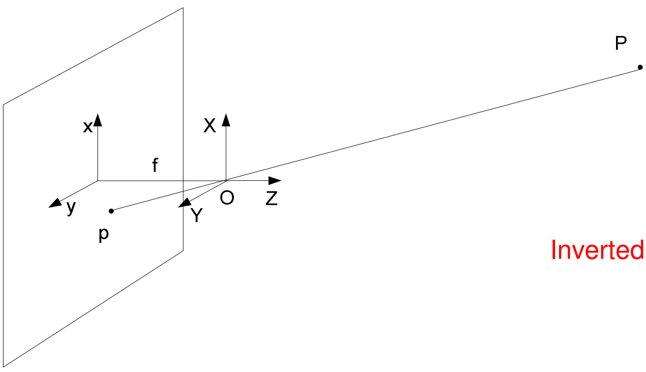
camera model



• arbitrary position

• internal parameters: focal length, scaa factors, principal point

pin-hole model



Inverted image

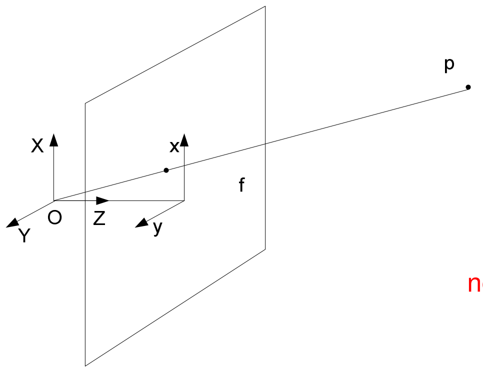
point in space: $X = [X\ Y\ Z]^T$

point in the image: $x = [x\ y]^T$

perspective projection

$$x = -f\,\frac{X}{Z} \qquad y = -f\,\frac{Y}{Z}$$

Perspective projection with frontal plane



non inverted image

point in space: $X = [X\ Y\ Z]^T$

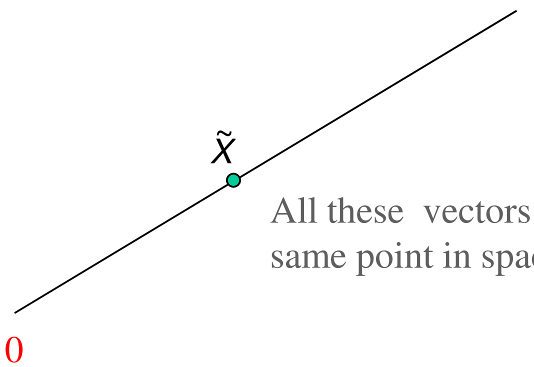
point in the image: $x = [x\ y]^T$

perspective projection

$$x = f\frac{X}{Z} \qquad y = f\frac{Y}{Z}$$

homogeneous coordinates

$$\tilde{X} = \alpha \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \qquad \tilde{x} = \beta \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \alpha, \beta \text{ arbitrary } (\alpha, \beta \neq 0)$$



All these vectors $\tilde{X}$ represent the same point in space

Perspective projection (ideal case)

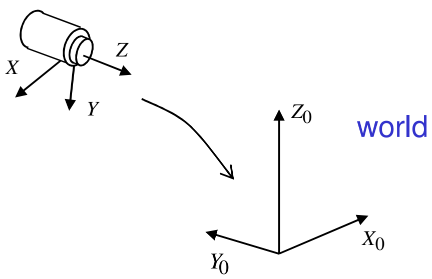
$$Z\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\Pi_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = [I \mid 0]$$

$$\lambda\tilde{x} = \Pi_0 \tilde{X}$$

extrinsic parameters

aamera in arbitrary position



$$X = RX_0 + t$$ Cartesian coordinates

$$\tilde{X} = g\tilde{X}_0$$ homogeneous coordinates

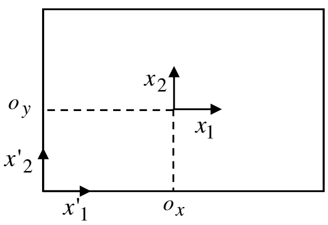
$$g = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}$$

$$\lambda\tilde{x} = \Pi_0 g \tilde{X}_0$$

$$\lambda\tilde{x} = [R \mid t]\tilde{X}_0$$

R,T são parâmetros extrínsecos da câmara

Intrinsic parameters



internal model:
conversion from metric coordinates to pixels

$$x' = fs_x x + o_x$$
$$y' = fs_y y + o_y$$

(x´,y´) in pixels

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} fs_x & 0 & o_x \\ 0 & fs_y & o_y \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = Kx$$

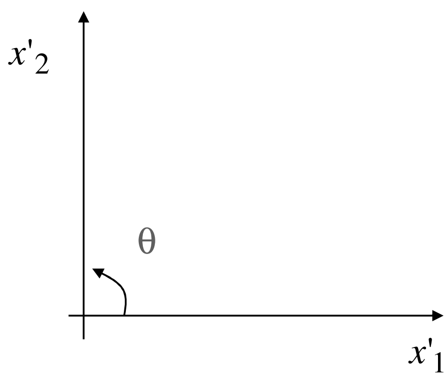
K upper triangular matrix

comment

Matrix K is usually considered as a upper triangular matriz without additional constraints

This is equivalent to assuming that the angle θ between the two coordinate axis can be slightly different from 90º.

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & c_1 \\ 0 & \dfrac{\beta}{\sin\theta} & c_2 \\ 0 & 0 & 1 \end{bmatrix}$$



Full perspective model

Camera model: $\tilde{x} = \Pi \tilde{X}$ $\Pi = K[R \mid T]$

Π is a 3×4 matrix denoted as camera matrix.

the camera model is linear in homogeneous coordinates!

Camera model in Cartesian equations

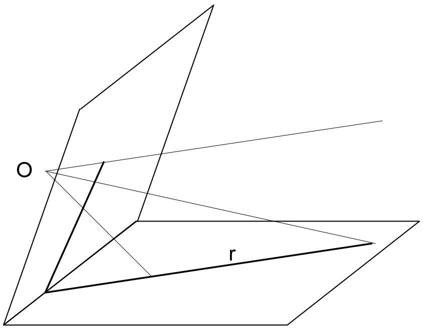
Cartesian coordinates

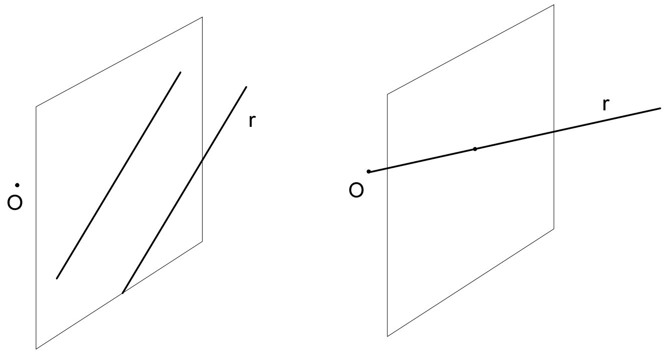
$$x_1 = \frac{\pi_1^T X}{\pi_3^T X} \quad , \quad x_2 = \frac{\pi_2^T \tilde{X}}{\pi_3^T \tilde{X}} \quad , \quad \tilde{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} \pi_1^T \\ \pi_2^T \\ \pi_3^T \end{bmatrix}$$

11 graus de liberdade

Matrix P é specified apart from a scale factor!

Projection of straight lines





general case

special cases

vanishing point

$$\tilde{X} = \tilde{X}_0 + \alpha \tilde{T}$$

$$\tilde{X} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \tilde{X}_0 = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \\ 1 \end{bmatrix} \quad \tilde{T} = \begin{bmatrix} T_x \\ T_y \\ T_z \\ 0 \end{bmatrix}$$

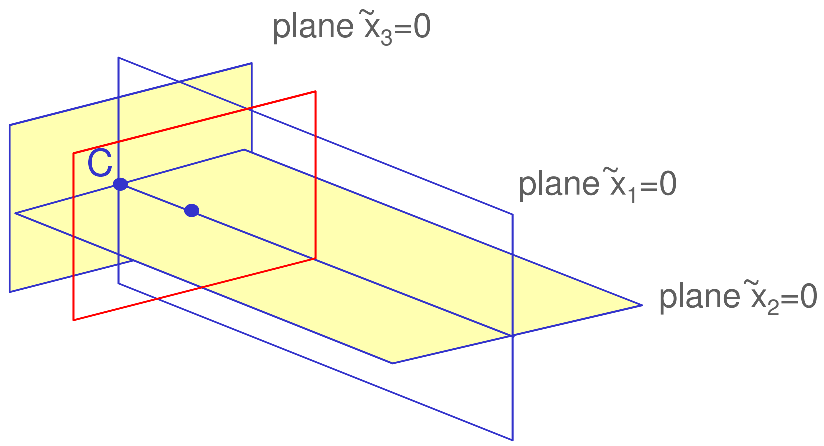
$$\tilde{x} = \Pi \tilde{X}_0 + \alpha \Pi \tilde{T}$$

When a goes to infinity

$$\tilde{x} = \Pi \tilde{T} \qquad x = \frac{\pi_1^T \tilde{T}}{\pi_3^T \tilde{T}} \qquad y = \frac{\pi_2^T \tilde{T}}{\pi_3^T \tilde{T}}$$

The vanishing point does not depend on X0. Each set of parallel lines has its on valishing point.

Optical center

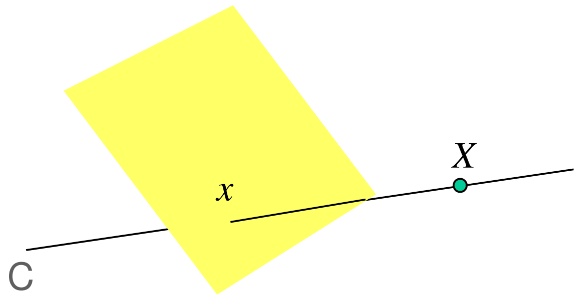


optical center

$$\Pi C = 0$$

optical ray

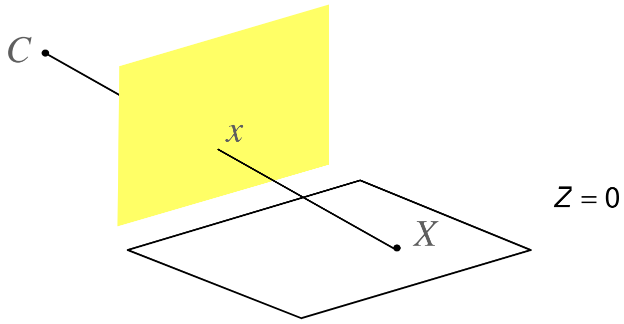
How to obtain the optical ray projected on *x* ?



This line is defined by 2 pontos e.g., the optical center C and

$$\tilde{X} = \Pi^T (\Pi \Pi^T)^{-1} \tilde{x}$$

Projective transformation from the plane Z=0



$$x = \frac{p_{11}X + p_{12}Y + p_{14}}{p_{31}X + p_{32}Y + p_{34}}$$

$$y = \frac{p_{21}X + p_{22}Y + p_{24}}{p_{31}X + p_{32}Y + p_{34}}$$

em coordenadas homogéneas

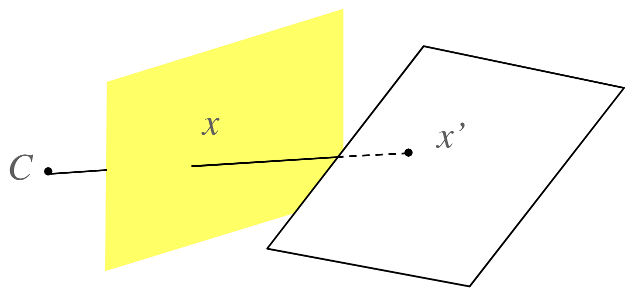
$$\boxed{\tilde{x} = H\ \tilde{x}'} \qquad \tilde{x}' = [X\ Y\ 1]^T$$

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$h_{ij} = p_{ij}\ ,\ i = 1,2\ h_{3j} = p_{4j}$$

The converse i also true $\tilde{x}' = H'\ \tilde{x}$

projective transformation from a plane

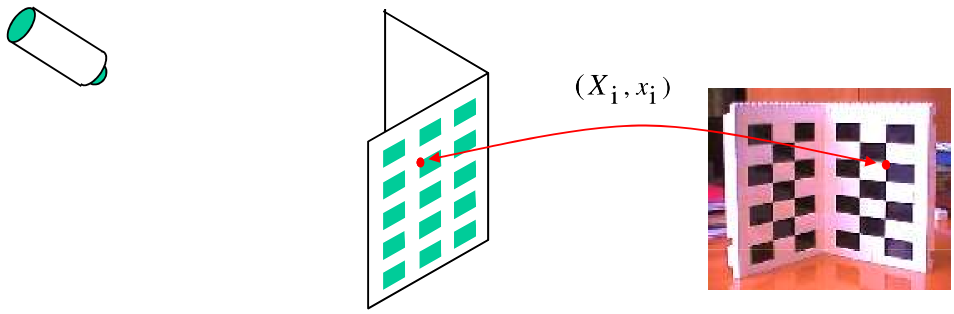


in homogeneous coordinates

$$\boxed{\tilde{x} = H\ \tilde{x}'}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

Camera calibration



Calibration involves the estimation of the intrinsic and extrinsic parameters *K, R, t* from experimental data.

data: we assume that we know several pairs of corresponding points in 3D and in the image plane

$$\{(X_i, x_i), i = 1, \ldots, n\} \qquad X_i \in R^3,\ x_i \in R^2$$

linear method

Projective model

$$x = \frac{\pi_1 \cdot \tilde{X}}{\pi_3 \cdot \tilde{X}} \qquad (\pi_1 - x\pi_3)\cdot \tilde{X} = 0$$

$$y = \frac{\pi_2 \cdot \tilde{X}}{\pi_3 \cdot \tilde{X}} \qquad (\pi_2 - y\pi_3)\cdot \tilde{X} = 0$$

Há um par de equações lineares que relacionam *x* e *X*.

Conhecendo n pares (X_i, x_i) obtém-se

$$\boxed{M\pi = 0} \qquad M = \begin{bmatrix} \tilde{X}^{1T} & 0 & -x^1 \tilde{X}^{1T} \\ 0 & \tilde{X}^{1T} & -y^1 \tilde{X}^{1T} \\ \vdots & \vdots & \vdots \\ \tilde{X}^{nT} & 0 & -x^n \tilde{X}^{nT} \\ 0 & \tilde{X}^{nT} & -y^n \tilde{X}^{nT} \end{bmatrix} \qquad \pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{bmatrix}$$

Linear method II

Minimize the norm of the residual $r = M\pi$ $|\pi|^2 = 1$

Solução: π é o vector próprio (unitário) associado ao menor valor próprio de $M^T M$.